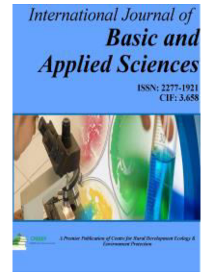
Content is available at: CRDEEP Journals
Journal homepage: http://www.crdeepjournal.org/category/journals/ijbas/

International Journal of Basic and Applied Sciences
(ISSN: 2277-1921) (Scientific Journal Impact Factor: 6.188)

UGC Approved-A Peer Reviewed Quarterly Journal



Full Length Research Paper

A Note on the Oscillation of Conformable Differential Equations

P.V.H. S Sai Kumar[1]
Department of Mathematics, Talla Padmavathi College of Engineering, Tekulagudem (Somidi), Kazipet, Warangal-506003,Telangana, India.

ARTICLE DETAILS

Corresponding Author:
P.V.H. S Sai Kumar

Key words:
Oscillation, Conformable Differential Equation

ABSTRACT

The aim of this paper is to study the oscillatory behavior of Conformable differential equations of the form

$$D_\alpha\left(b(t)D_\alpha\left(a(t)D_\alpha y(t)\right)\right)+q(t)y(t)=0$$

Where; D_α denotes the conformable derivative of order α with $0<\alpha\le 1$.By using the generalized Riccati transformation technique, new sufficient conditions for oscillations of solutions are established.

1. Introduction

In this paper, we investigate the oscillatory behavior of solutions of conformable differential equation of the form

$$D_\alpha\left(b(t)D_\alpha\left(a(t)D_\alpha y(t)\right)\right)+q(t)y(t)=0 \quad (1)$$

where D_α denotes the conformable derivative of order α with $0<\alpha\le 1$.

If $\alpha=1$, then we have the following third order differential equation

$$\left(b(t)\left(a(t)y'(t)\right)'\right)'+q(t)y(t)=0$$

We study the oscillation of equation (1) under the following two cases

$$\int_{t_0}^{\infty}\frac{1}{a(s)}d_\alpha(s,t_0)=\infty, \qquad \int_{t_0}^{\infty}\frac{1}{b(t)}d_\alpha(s,t_0)=\infty. \qquad (2)$$

Fractional derivative originated in the 17 th century at the time of Newton and Leibniz. In 1965 L' Hospital introduced the concept of Fractional Derivatives. Fractional calculus is the study of derivatives and Integrals of non-Integer order and is the Generalized form of classical derivatives and Integrals. Fractional differential equations have gained importance during the past few years. Fractional differential equations occur in many research fields such as in modeling mechanical and electrical properties, they have also been applied to study several physical phenomena such as Chemistry and Physics.

We observe that the oscillatory behavior of solutions of fractional differential equations have been studied by many authors. For example, we refer to [6,19,21] and the monographs [3,4,5,15]. We noticed that very little attention is paid to oscillation of linear / non-linear conformable fractional differential equations [17,18]. We also see that there are many papers which have discussed about the conformable fractional derivative see for example [1,2,8,12,14,]. Significant attention has been received

[1]Author can be contacted at: *Department of Mathematics, Talla Padmavathi College of Engineering, Tekulagudem (Somidi), Kazipet, Warangal-506003,Telangana, India.*

Received: 30-October-2024; Sent for Review on: 5-Nov-2024; Draft sent to Author for corrections: 12-Nov-2024; Accepted on: 18-Nov-2024; Online Available from 20-Nov-2024
DOI: 10.13140/RG.2.2.25823.47528

on the oscillation of Fractional differential equations as a new Research. In 2014 Khalil et al. in [11] introduced a new differential operator called the conformable derivative.

In 2016, Jessada Tariboon and Sotiris K. Ntouyas [9] established the oscillation results for the solutions of impulsive conformable fractional differential equations of the form

$$\{ {}_{t_k}T^k(p(t)[{}_{t_k}T^{\alpha}x(t) + r(t)x(t)]) + q(t)x(t) = 0, \;\; t \geq t_0, \;\; t \neq t_k$$
$$x(t_k^{+}) = a_k\, x(t_k^{-}), \quad {}_{t_k}T^{\alpha}\, x(t_k^{+}) = b_k\, {}_{t_{k-1}}T^{\alpha}x(t_k^{-}), \qquad k = 1,2 \ldots$$

and some new oscillation results are obtained by using the equivalence transformation and by applying Riccati techniques.
In 2019, A. Ogunbanjo & P. Arawomo [14] studied the oscillation of solutions to the generalized forced nonlinear conformable fractional differential equation of the form

$$D_{\alpha}[a(t)\psi(x(t))D_{\alpha}x(t)] + P(t, x(t), D_{\alpha}x(t)) = Q\,(t, x(t), D_{\alpha}x(t)) \quad t \geq t_0 > 0,$$

with $D_{\alpha}(.)$ denotes the operator called conformable fractional derivative of order α and C^{α} denotes continuous function with fractional derivative of order α and $a \in C^{\alpha}[[t_0,\infty),\mathbb{R}]$ and $P, Q \in C^{\alpha}[[t_0,\infty),\mathbb{R}^2,\mathbb{R}]$.

A nontrivial solution of $y(t)$ of differential equation (1) is said to be oscillatory if it has arbitrarily large zeros, otherwise non oscillatory. The equation (1) is oscillatory if all its solutions are oscillatory. In this paper, we establish several oscillation criteria for conformable fractional differential equations by applying Riccati transformation technique. These results are considered essentially new.

2. Method

In this paper we use few Lemmas and definition which are helpful to prove our results.

3. Main Results

We assume that (H_1) $b(t), a(t) \in C^{'}([t_0,\infty),\mathbb{R}_+)$ and $q(t) \in C([t_0,\infty),\mathbb{R}_+)$; (H_2) $y(t)$ and $y^{'}(t)$ are differentiable functions on $\mathbb{R}$.

Definition 1: [17] The conformable derivative from t_0 of a function f:[$t_0,\infty) \to \mathbb{R}$ of order α with $0 < \alpha \leq 1$ and $t \geq t_0 > 0$ is defined by

$$D_{\alpha}f(t) = \lim_{\varepsilon\to 0}\frac{(f(t+\varepsilon(t-t_0)^{1-\alpha}) - f(t)}{\varepsilon}.$$

Lemma 3.1: [7] Let f and g be $\alpha -$ differentiable at a point $t \geq t_0$ and $\alpha \in (0,1]$

Then
$(i) D_{\alpha}(\lambda) = 0$, where λ is a constant.
$(ii) D_{\alpha}(t^p) = pt^{p-\alpha}$, for p.
$(iii) D_{\alpha}(fg) = fD_{\alpha}(g) + gD_{\alpha}f$.
$(iv) D_{\alpha}\left(\frac{f}{g}\right) = \frac{gD_{\alpha}(f) - fD_{\alpha}(g)}{g^2}$.
$(v) D_{\alpha}(af + bg) = aD_{\alpha}(f) + bD_{\alpha}g$ for all real constants a, b.
(vi) If f is differentiable, then $D_{\alpha}f(t) = t^{1-\alpha}f^{'}(t)$.

Lemma 3.2 [13]: Let $\alpha \in (0,1]$.Then the conformable integral of order α starting at t_0 is defined by

$$I_{\alpha}f(t) = \int_{t_0}^{t}(s-t_0)^{\alpha-1}f(s)ds = \int_{t_0}^{t}f(s)d_{\alpha}(s,t_0).$$

If the conformable integral of a given function f exists, we call that f is α- integrable.

Lemma 3.3 [13]: If $\alpha \in (0,1]$ and $f \in C^{'}([t_0,\infty),\mathbb{R})$, then for all $t > t_0$, we have $I_{\alpha}(D_{\alpha}f(t)) = f(t) - f(t_0)$ and $D_{\alpha}(I_{\alpha}f(t)) = f(t)$.

Lemma 3.4. [16]: We introduce a class of functions Υ in this paper. We say that a function $\phi(t,s,l)$ belongs to the function class Υ denoted by $\emptyset \in \Upsilon$, if $\emptyset \in C(E,R)$, where $E = \{(t,s,l): t_0 \leq l \leq s \leq t < \infty\}$ which satisfies $\emptyset(t,t,l) = 0$, $\emptyset(t,l,l) > 0 \emptyset(t,s,l) \neq 0$ for $l < s < t$ and has the partial derivative $\frac{\partial_{\alpha}\emptyset}{\partial s_{\alpha}}$ on E defined by

$$\frac{\partial_{\alpha}\emptyset\,(t,s,l)}{\partial s_{\alpha}} = \emptyset(t,s,l)\emptyset(t,s,l), \text{ where } \emptyset \in \Upsilon. \qquad (3)$$

Further we define the operator

$$A(g;l,t) = \int_l^t \emptyset^2 (t,s,l) g(s) d_\alpha(s,t_0) \quad (4)$$

for $t \geq s \geq l \geq t_0$ where $g(s) \in C[t_0,\infty)$.

It is easy to verify that linear operator $A(g_\alpha : l, t) = -2A(\emptyset g; l, t)$ for $g \in C_\alpha([0,\infty),\mathbb{R})$. (5)

Lemma 3.5 [20]: Set $B(v) = b_1(\xi)v - b_2(\xi)v^{\frac{\gamma+1}{\gamma}}$, where the function $b_1(\xi)$ is arbitrary function $b_2(\xi)$ and the function v are always positive, γ is a positive constant .Then the function $B(v)$ has the maximum value B_{max} at v_0 on R such that

$$B(v) \leq B_{max} = \frac{\gamma^\gamma}{\gamma+1} \frac{{b_1}^{\gamma+1}(\xi)}{{b_2}^{\gamma}(\xi)}$$

Where $v_0 = \left(\frac{\gamma}{\gamma+1}\frac{b_1(\xi)}{b_2(\xi)}\right)^\gamma$.

Lemma 3.6: Let

$$t^{1-\alpha}x(t) \leq (t-T)^3 \psi'(t)\, a(t) D^\alpha x(t)$$

and suppose $\psi'(t) \leq M$

Then,

$$\frac{x(t)}{a(t)D_\alpha x(t)} \leq \frac{(t-T)^3 M}{t^{1-\alpha}}$$

Theorem 3.1: Let the conditions (H_1) to (H_2) and (2) hold. Assume that for each $l \in C_\alpha([t_0,\infty),\mathbb{R})$ there exists a function $\emptyset \in y$ such that

$$\lim_{\xi\to\infty} sup\, A\left[\tilde{\rho}\tilde{q}\frac{(\xi-T)^3 M}{\xi^{1-\alpha}} - \left(\frac{\frac{\tilde{\rho}'}{\tilde{\rho}}+2\varphi}{\gamma+1}\right)^{\gamma+1}(\tilde{\rho}\tilde{a})^\gamma, \xi_1, \xi\right] > 0. \quad (6)$$

where A and $\emptyset$ are introduced in (3) and (4) respectively. Then every solution of (1) is oscillatory.

Proof: Let $y(t)$ be a nonoscillatory solution of equation (1) Without loss of generality we may assume that $y(t) > 0\ t \geq t_1$, where t_1 is chosen so large that Lemma (3.6) hold.

Define the function $\omega(t)$ by

$$\omega(t) = \rho(t)\frac{b(t)D_\alpha\big(a(t)D_\alpha y(t)\big)}{a(t)D_\alpha y(t)}, \quad t \geq t_1 \quad (7)$$

Then $\omega(t) > 0$ and

$$D_\alpha\omega(t) = \frac{b(t)D_\alpha(a(t)D_\alpha y(t))}{a(t)D_\alpha y(t)}D_\alpha\rho(t) + \rho(t)D_\alpha\left(\frac{b(t)D_\alpha(a(t)D_\alpha y(t))}{a(t)D_\alpha y(t)}\right)$$

$$D_\alpha\omega(t) = \frac{b(t)D_\alpha(a(t)D_\alpha y(t))}{a(t)D_\alpha y(t)}D_\alpha\rho(t) + \rho(t)\left(\frac{a(t)D_\alpha y(t)D_\alpha\Big(b(t)D_\alpha\big(a(t)D_\alpha y(t)\big)\Big) - b(t)D_\alpha(a(t)D_\alpha y(t))D_\alpha(a(t)D_\alpha y(t))}{(a(t)D_\alpha y(t))^2}\right)$$

$$D_\alpha\omega(t) = \frac{b(t)D_\alpha(a(t)D_\alpha y(t))}{a(t)D_\alpha y(t)}D_\alpha\rho(t) + \rho(t)\frac{D_\alpha\Big(b(t)D_\alpha\big(a(t)D_\alpha y(t)\big)\Big)}{a(t)D_\alpha y(t)} - \frac{b(t)D_\alpha(a(t)D_\alpha y(t))D_\alpha(a(t)D_\alpha y(t))}{(a(t)D_\alpha y(t))^2}$$

$$D_\alpha\omega(t) = \frac{D_\alpha\rho(t)}{\rho(t)}\omega(t) - \frac{\rho(t)q(t)y(t)}{a(t)D_\alpha y(t)} - \frac{b(t)D_\alpha(a(t)D_\alpha y(t))D_\alpha(a(t)D_\alpha y(t))}{(a(t)D_\alpha y(t))^2}$$

From Lemma 3.6 and (7) we get,

$$D_\alpha \omega(t) = \frac{D_\alpha \rho(t)}{\rho(t)}\omega(t) - \rho(t)q(t)\frac{(t-T)^3 M}{t^{1-\alpha}} - \frac{1}{\rho^2(t)b(t)}\omega^2(t) \qquad (8)$$

Let $\omega(t) = \tilde{\omega}(\xi)$. Then $D_\alpha \omega(t) = \tilde{\omega}'(\xi)$ and $D_\alpha \rho(t) = \tilde{\rho}'(\xi)$, so the above inequality becomes

$$\tilde{\omega}'(\xi) \le \frac{\tilde{\rho}'(\xi)}{\tilde{\rho}(\xi)}\tilde{\omega}(\xi) - \tilde{\rho}(\xi)\tilde{q}(\xi)\frac{(t-T)^3 M}{t^{1-\alpha}} - \frac{1}{\tilde{\rho}^2(\xi)\tilde{b}(\xi)}\tilde{\omega}^2(\xi) \qquad (9)$$

Applying the operator $A[\,.;\ \xi_1,\xi\,](\xi \ge \xi_1)$ to (9) we note that $A[\,.;\ \xi_1,\xi\,]$ is a linear operator, then we get

$$A[\omega';\xi_1,\xi] \le A\left[\frac{\tilde{\rho}'(\xi)}{\tilde{\rho}(\xi)}\tilde{\omega}(\xi) - \frac{1}{\tilde{\rho}^2(\xi)\tilde{b}(\xi)}\tilde{\omega}^2(\xi);\xi_1,\xi\right] - A\left[\tilde{\rho}(\xi)\tilde{q}(\xi)\frac{(\xi-T)^3 M}{\xi^{1-\alpha}},\xi_1,\xi\right]$$

By (5) and the above inequality, we get

$$A\left[\tilde{\rho}\tilde{q}\frac{(\xi-T)^3 M}{\xi^{1-\alpha}},\xi_1,\xi\right] \le A\left[\left(\frac{\tilde{\rho}'}{\tilde{\rho}} + 2\varphi\right)\omega - \frac{1}{\tilde{\rho}\tilde{b}}\tilde{\omega}^2;\xi_1,\xi\right]$$

To use the Lemma 3.5,
Set

$$b_1 = \frac{\tilde{\rho}}{\tilde{\rho}} + 2\varphi, \qquad b_2 = \frac{1}{\tilde{\rho}\tilde{b}}, \qquad v = \omega.$$

We can see that

$$A\left[\tilde{\rho}\tilde{q}\frac{(\xi-T)^3 M}{\xi^{1-\alpha}},\xi_1,\xi\right] \le A\left[\tilde{\rho}\tilde{q}\frac{(\xi-T)^3 M}{\xi^{1-\alpha}} - \left(\frac{\frac{\tilde{\rho}'}{\tilde{\rho}} + 2\varphi}{\gamma+1}\right)^{\gamma+1}(\tilde{\rho}\tilde{b})^{\gamma},\xi_1,\xi\right]$$

This means that

$$A\left[\tilde{\rho}\tilde{q}\frac{(\xi-T)^3 M}{\xi^{1-\alpha}} - \left(\frac{\frac{\tilde{\rho}'}{\tilde{\rho}} + 2\varphi}{\gamma+1}\right)^{\gamma+1}(\tilde{\rho}\tilde{a})^{\gamma},\xi_1,\xi\right] \le 0$$

Taking the lim sup in the above inequality, we get

$$\lim_{\xi\to\infty} sup\ A\left[\tilde{\rho}\tilde{q}\frac{(\xi-T)^3 M}{\xi^{1-\alpha}} - \left(\frac{\frac{\tilde{\rho}'}{\tilde{\rho}} + 2\varphi}{\gamma+1}\right)^{\gamma+1}(\tilde{\rho}\tilde{a})^{\gamma},\xi_1,\xi\right] \le 0 \qquad (10)$$

which is a contradiction to (6). Therefore equation (1) is oscillatory.

Next, we present some new oscillation results for equation (1), by using integral averages condition of Philos- type. First, we introduce a class of functions $\mathbb{R}$. Let

$$D_0 = \{(t,s): t > s \ge t_0\} \text{ and } D = \{(t,s): t \ge s \ge t_0\}.$$

The function H is said to belong to the class $\mathbb{R}$ if

(i) $H(t,t) = 0$ for $t \ge t_0$; $H(t,s) > 0$ for $(t,s) \in D_0$;

(ii) H has a continuous and nonpositive partial derivative on D_0 with respect to the second variable such that

$$h(t,s) = H(t,s)\frac{\rho'(s)}{\rho(s)} + \frac{\partial H}{\partial s}(t,s)$$

where h is a suitable function $\omega(t)$.

Theorem 3.2: Assume that (2) &$(A_1) - (A_2)$ hold. Further there exists $\rho \in C_\alpha([t_0,\infty),\mathbb{R}_+)$ and $H \in \mathbb{R}$, such that

$$\lim_{\xi\to\infty} sup \frac{1}{H(\xi,\xi_1)}\int_{\xi_1}^{\xi}\left(H(\xi,s)\tilde{q}(s)\,\tilde{\rho}(s)\,\frac{(s-T)^3M}{s^{1-\alpha}}-\int_{\xi_1}^{\xi}\frac{1}{4}\frac{\tilde{\rho}^2(s)\tilde{b}(s)}{H(\xi,s)}h^2(\xi,s)\right)d_\alpha(s,\xi_0)=\infty. \quad (11)$$

Then every solution of system (1) is oscillatory.

Proof.: In this case we define again the function $\omega(t)$ by (7) and proceeding as in the proof of Theorem (3.1) to obtain (9). Then by substituting ξ with s in (9) and multiplying both sides by $H(\xi,s)$ and integrating from ξ_1 to ξ, we have

$$\int_{\xi_1}^{\xi}H(\xi,s)\tilde{q}(s)\tilde{\rho}(s)\frac{(s-T)^3M}{s^{1-\alpha}}d_\alpha(s,\xi_0)\le\int_{\xi_1}^{\xi}H(\xi,s)\frac{\tilde{\rho}'(s)}{\tilde{\rho}(s)}\tilde{\omega}(s)d_\alpha(s,\xi_0)$$

$$-\int_{\xi_1}^{\xi}H(\xi,s)\tilde{\omega}'(s)d_\alpha(s,\xi_0)-\int_{\xi_1}^{\xi}H(\xi,s)\frac{1}{\tilde{\rho}^2(s)\tilde{b}(s)}\tilde{\omega}^2(s)\,d_\alpha(s,\xi_0)$$

$$\le H(\xi,\xi_1)\tilde{\omega}(\xi_1)+\int_{\xi_1}^{\xi}\left(H(\xi,s)\frac{\tilde{\rho}'(s)}{\tilde{\rho}(s)}+\frac{\partial H}{\partial s}(\xi,s)\right)\tilde{\omega}(s)\,d_\alpha(s,\xi_0)-\int_{\xi_1}^{\xi}H(\xi,s)\frac{1}{\tilde{\rho}^2(s)\tilde{b}(s)}\tilde{\omega}^2(s)d_\alpha(s,\xi_0)$$

$$\le H(\xi,\xi_1)\tilde{\omega}(\xi_1)+\int_{\xi_1}^{\xi}\tilde{\omega}(s)h(\xi,s)\,d_\alpha(s,\xi_0)-\int_{\xi_1}^{\xi}H(\xi,s)\frac{1}{\tilde{\rho}^2(s)\tilde{b}(s)}\tilde{\omega}^2(s)d_\alpha(s,\xi_0)$$

$$\le H(\xi,\xi_1)\tilde{\omega}(\xi_1)+\int_{\xi_1}^{\xi}\frac{1}{4}\frac{\tilde{\rho}^2(s)\tilde{b}(s)}{H(\xi,s)}h^2(\xi,s)d_\alpha(s,\xi_0)$$

From this we conclude that

$$\int_{\xi_1}^{\xi}\left(H(\xi,s)\tilde{q}(s)\tilde{\rho}(s)\,\frac{(s-T)^3M}{s^{1-\alpha}}-\int_{\xi_1}^{\xi}\frac{1}{4}\frac{\tilde{\rho}^2(s)\tilde{b}(s)}{H(\xi,s)}h^2(\xi,s)\right)d_\alpha(s,\xi_0)\le H(\xi,\xi_1)\tilde{\omega}(\xi_1)$$

Dividing the above inequality by $H(\xi,\xi_1)$ we get,

$$\frac{1}{H(\xi,\xi_1)}\int_{\xi_1}^{\xi}\left(H(\xi,s)\tilde{q}(s)\tilde{\rho}(s)\frac{(s-T)^3M}{s^{1-\alpha}}-\int_{\xi_1}^{\xi}\frac{1}{4}\frac{\tilde{\rho}^2(s)\tilde{b}(s)}{H(\xi,s)}h^2(\xi,s)\right)d_\alpha(s,\xi_0)\le\tilde{\omega}(\xi_1)$$

Letting $\xi\to\infty$,

$$\lim_{\xi\to\infty} sup \frac{1}{H(\xi,\xi_1)}\int_{\xi_1}^{\xi}\left(H(\xi,s)\tilde{q}(s)\,\tilde{\rho}(s)\,\frac{(s-T)^3M}{s^{1-\alpha}}-\int_{\xi_1}^{\xi}\frac{1}{4}\frac{\tilde{\rho}^2(s)\tilde{b}(s)}{H(\xi,s)}h^2(\xi,s)\right)d_\alpha(s,\xi_0)\le\tilde{\omega}(\xi_1).$$

which contradicts (11). Then every solution of (1) is oscillatory.

4. Conclusion

In this paper, the Author discussed the oscillatory behavior of solutions to Conformable differential equations. By using the classical Riccati technique and the properties of conformable derivative some new oscillation criteria for equation (1) are established. It can be seen that this approach can also be applied to the oscillation of other fractional differential equations with more complicated forms involving modified Riemann-Liouville derivative.

5. References

[1]D. RAmderson and D.J.Ulness, (2015), Newly defined conformable derivatives, Advances in Dynamical Systems and Applications, 10(2),109-137.

[2] T. Abdeljawad, (2015), On conformable fractional calculus, Journal of computational and Applied Mathematics, Vol.279, 57-66.

[3]K. Diethelm, (2010), The Analysis of Fractional Differential Equations, Vol.2004, Springer, Berlin, Germany.

[4] V. Daftardar-Gejji, (2014) Fractional calculus theory and Applications, Narosa publishing house Pvt. Ltd.

[5] S. Das,(2008), Functional Fractional Calculus for System Identification and Controls, Springer, Berlin.

[6] Grace, S.R., Agarwal, R.P., Wong, P.J.Y. and Zafer, A, (2012),On the Oscillation of Fractional Differential equations, Fractional Calculus and Applied Analysis, 15, 222-23.

[7]George E. Chatzarakisy, Kandasamy Logaarasiz, Thangaraj Raja, Vadivel Sadhasivam,(2019), Interval Oscillation Criteria for Conformable Fractional Differential Equations with Impulses, Applied Mathematics E-Notes, Vol.19, 354-369.

[8]V Ganesan, M Sathish Kumar, (2016),Oscillation Theorems for Fractional Order Neutral Differential Equations,2(10),46-51.
[9]JessadaTariboon and Sotiris K. Ntouyas, (2016) Oscillation of impulsive conformable fractional differential equations, Open Math.14: 497–508.
[10]A. A Kilbas, H.M Srivastava, J. J Trujillo, (2006), Theory and Applications of Fractional Differential equations. Elsevier, Amsterdam, Vol.204, 1-523.
[11]R Khalil, M. Al Horani, A. Yousef and M. Sabaheh,(2014), A new definition of fractional derivative, J. Comput. Appl. Math., Vol.264, 65-70.
[12]T.U Khan, M.A Khan, (2019), Generalized conformable fractional operators. J.Comput.Appl. Math: Vol.346, 378-389.
[13] Limei Feng and ShurongSun, (2019) Oscillation theorems for three classes of conformable fractional differential equations, 2019:313,1-30.
[14]A. Ogunbanjo, P. Arawomo, (2019), Oscillation of solutions to a generalized forced nonlinear conformable Fractional differential equation, Proyecciones (Antofagasta. Online), Vol.38, 429-445.
[15]I. Podlubny, (1999), Fractional Differential Equations, Vol. 198, Academic Press, San Diego, Calif, USA, View at: Math Sci Net.
[16]Run Xu, Fanwei Meng, (2007), New Kamenev-type oscillation criteria for second order neutral nonlinear differential equations, Applied Mathematics and Computation, Vol.188, 1364–1370.
[17] J. Shao & Z. Zheng, (2019), Kamenev Type Oscillatory criteria for Linear Conformable Differential Equations, Discrete Dynamics in Nature and Society, Vol. 2019,1-10.
[18]Taher S. Hassan, (2009), Oscillation of third order nonlinear delay dynamic equations on time scales, Mathematica land Computer Modelling Vol.49, 1573–1586.
[19]Xiang, S., Han, Z., Zhao, P. and Sun, Y. (2014) Oscillation Behavior for a Class of Differential Equation with Fractional-Order Derivatives. Abstract and Applied Analysis, Article ID: 419597.
[20] Yakun Wang, Fanwei Meng and Juan Gu, (2021), Oscillation criteria of third-order neutral differential equations with damping and distributed deviating arguments, 2021:515,1-15.
[21]Zheng, B. (2013) Oscillation for a Class of Nonlinear Fractional Differential Equations with Damping Term Journal of Advanced Mathematical Studies, Vol.6,107-115.